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Advanced Statistical Physics - Problem set 4

Summer Terms 2022

Hand in: Hand in tasks marked with * to mailbox no. (43) inside ITP room 105b by Friday 06.05. at 9:15 am.

6. The Gardner Analysis* *3+3+2 Points*

(a) Determine the asymptotic behavior of the quantities describing Gibbs learning for large training set size α. To this end show starting from

$$\frac{R}{\sqrt{1-R}} = \frac{\alpha}{\pi} \int \boldsymbol{D}t \, \frac{\exp(-Rt^2/2)}{H(-\sqrt{R}t)}$$

that

$$1 - R \sim \left[\frac{\alpha}{\pi} \int \boldsymbol{D}t \, \frac{\exp(-t^2/2)}{H(-t)}\right]^{-2} \sim \frac{1.926}{\alpha^2}$$

Hint : You may use numerical methods to compute the final result.

(b) Use the result you obtain from part (a) to show that

$$\varepsilon \sim \frac{\sqrt{2}}{\int \boldsymbol{D}t \, [\exp(-t^2/2)]/H(-t)} \frac{1}{\alpha} \sim \frac{0.625}{\alpha},$$

as follows from

$$\varepsilon = \frac{1}{\pi}\arccos(R)$$

(c) Show also that

$$S \sim \frac{1}{2}\ln(1-R) \sim -\ln(\alpha)$$

7. Random Numbers Distribution *4+2 Points*

Generate numerically M random numbers x each being the product of N independent random numbers equally distributed between 1 and 2 with M between 10^3 and 10^6 and N between 5 and 50.

(a) Approximate the distribution of x by a histogram and compare the evolution with N of the most probable value x_{mp} of x with its average $\langle\langle x\rangle\rangle$ and the typical x value defined as $x_{typ} := \exp(\langle\langle \ln x\rangle\rangle)$

(b) Under what conditions in the asymptotic limit (for $N \to \infty$), should the most probable value and x_{typ} should coincide?